

# Self-Similarity in Particle Production in Pb-Pb Collisions at 158 AGeV

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**Abstract** – In present work, we use the method of scaled factorial moments to analyze pseudorapidity fluctuations in nucleus-nucleus collisions. The analysis is done on photon distributions obtained using preshower photon multiplicity detector. Scaled factorial moments are used to study short range fluctuations in pseudorapidity distributions of photons. Scaled factorial moments are calculated using horizontal corrected and vertical analysis. The results are compared with simulation analysis using VENUS event generator. The essence of experimental ultra-relativistic heavy ion collision physics is the production and study of strongly interacting matter at extreme energy densities, temperatures and consequent search for equation of state of nuclear matter. For the present analysis, data from the Photon Multiplicity Detector (PMD) is used. The focus of the analysis has been to examine pseudo-rapidity distributions obtained for the  $\gamma$ -like particles in pre-shower photon multiplicity detector. We also attempt to model the fluctuations seen in the data using a simple multi-source model. This allows the extension of scaled factorial moment analysis to bin sizes smaller than those accessible to other experimental techniques. For a sample of 15000 central collisions, moments are calculated using both horizontal and vertical analysis techniques.

**Index Terms** – Pseudorapidity; Scaled Factorial Moments; Heavy Ion; VENUS event generator; Nuclear Matter.

## 1. INTRODUCTION

An important feature of central ultrarelativistic heavy ion collisions is the presence of significant fluctuations in rapidity density. This phenomenon has been seen in both cosmic ray experiments and in accelerator experiments [1]. Data from WA98 experiment as well as heavy ion collisions exhibit such a power law behavior as  $\delta y$  is decreased. This behaviour, in which the moments increase as the size of the interval is decreased, is called intermittency. The word “intermittency” originally came from the field of fluid-dynamics. In an isotropic turbulent fluid of a high Reynold number, an intermittent structure appears as tube-like regions of a high vorticity isosurface.. This stochastic, irregular behavior leads to a similar power-law variations of the moments as the size of region is decreased. The properties of such fluctuations have been extensively measured and discussed in the turbulence of fluids. Ultra-relativistic heavy ion collisions (URHICs) carried out at the Large Hadron Collider (LHC) at CERN and the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory create super-hot and dense matter. The observed thermal particle spectra and anisotropic collective

flow seen in such collisions has been taken as evidence of the creation of a deconfined high-temperature quark-gluon plasma (QGP). The existence of the QGP itself is a prediction of finite-temperature quantum-chromodynamics (QCD), and the QGP is believed to have been the state of the early universe. Comparisons between theory and experiment suggest that LHC ,URHICs produce a QGP with an initial temperature on the order of 500–600 MeV for 2.76 TeV Pb-Pb collisions. The LHC has recently performed URHICs with nucleon–nucleon center-of-mass energies of 5.023 TeV, which are expected to produce initial temperatures on the order of 600–700 MeV. This can have an impact on the various signatures for QGP formation, such as heavy quarkonium suppression.

## 2. MATHEMATICAL FORMULATION

The phase-space distribution of hadrons produced in high-energy collisions has been used for many years as a tool to investigate the elementary mechanisms governing such reactions. Models incorporating perturbative QCD for hard (high-momentum transfer) scattering and semiphenomenological formulations for soft (low-momentum transfer) collisions and hadronization have met with great success in single-particle distributions in systems as simple as  $e^+e^-$  and as complex as nucleus-nucleus collisions [2]. In addition, large phase-space scale multi-particle distributions in collisions dominated by hard processes (jets) are also well described by such models. In recent years, interest has grown in the investigation of small phase-space scale multi-particle distribution. The initial impetus for this came from the study of high-energy nucleus-nucleus collisions in connection with a possible phase transition from ordinary hadronic matter to a quark-gluon plasma [3]. However, it was soon realized that such a detailed study of multi-particle distributions in simpler collisions may also yield new information on elementary particle production mechanisms, possibly relating to their fractal properties. Thus, there are two quite separate goals for the current study of multi-particle production : (i) the investigation of elementary particle production mechanisms, using simple probes, and (ii) the search for collective phenomena, usually using complex probes such as heavy nuclei. The strategy to identify collective phenomena is to search for deviations from the multiparticle distributions predicted by a simple superposition of elementary sources.

Measurements of multi-particle distributions require great care to interpret because of the unavoidable fluctuations due to finite particle multiplicity, resonance production, and detector effects such as interactions with material and limited two-track resolution. Bialas and Peschanski [4] suggested a means of suppressing the fluctuations due to finite multiplicity by calculating the mean scaled factorial moments  $\langle F_q \rangle$  of the multiplicity distribution. Given a total interval of rapidity  $\Delta y$  divided into M equal bins of size  $\delta y = \frac{\Delta y}{M}$ , the mean scaled factorial moments  $\langle F_q \rangle$  of order q is defined as

$$\langle F_q \rangle = M^{q-1} \sum_{m=1}^M \frac{K_m(K_m-1)\dots(K_m-q+1)}{N(N-1)\dots(N-q+1)} \quad (1)$$

Where  $K_m$  denotes the number of particles in mth bin. The behavior of  $F_q$  as a function of  $\delta y$  is an indicator of the correlation length for fluctuations.  $\langle \dots \rangle$  indicates an average over events, and  $\langle n \rangle$  is the mean multiplicity within  $\delta y$ . The dynamics of the particle production mechanism are then reflected in the dependence of  $\langle F_q \rangle$  on  $\delta y$ . In particular, a mechanism with a self-similar (“branching”) structure would exhibit a power-law dependence

$$\langle F_q \rangle \propto \delta y^{-\phi_q} \quad (2)$$

This power-law dependence is known as intermittency, and the general study of the dependence of  $\langle F_q \rangle$  on  $\delta y$  has come to be known by that name. The slope in a plot of  $\ln \langle F_q \rangle$  vs  $-\ln \delta y$  is  $\phi_q$ . Bialas and Peschanski [5] proposed that particle production in a longitudinally expanding fluid of quark-gluon plasma has an underlying branching structure in rapidity of final-state hadrons (intermittency in the multiplicity distribution). Others have suggested that the occurrence of intermittency is a signal of a second-order phase transition. However, more elementary particle production mechanisms having a self-similar cascading structure, such as the fragmentation of strings [or high-energy jets are also expected to produce intermittent final-state distributions. The analysis of scaled factorial moments has served as a sensitive statistical tool to compare particle production data. The hope is that, after accounting for all experimental effects, differences between models and data will point to new physics. There have been extensive experimental investigations of intermittency in the last few years. For the case of  $e^+e^-$  collisions, almost all studies find agreement in detail between data and commonly used particle production models. The situation with hadronic probes is much less clear.

The moments can either be normalized to the whole event sample or to the individual events. The moments are studied as a function of the chosen  $\eta$ -bin and the variation of the moments with varying  $\delta \eta$  may indicate an intermittent behaviour. Horizontal Analysis

$$\langle F_q \rangle_H = \frac{1}{N_{evts}} \sum_{i=1}^{N_{evts}} M^{q-1} \sum_{m=1}^M \frac{K_{m,i}(K_{m,i}-1)\dots(K_{m,i}-q+1)}{\langle N \rangle^q} \quad (3)$$

Where  $N_{evts}$  is the number of events in the sample,  $K_{m,i}$  is the content of bin m in event I and  $\langle N \rangle$  is the average multiplicity in pseudorapidity window  $\Delta \eta$ . Vertical analysis

$$\langle F_q \rangle_V = \frac{1}{M} \sum_{m=1}^M \frac{1}{N_{evts}} \sum_{i=1}^{N_{evts}} \frac{K_{m,i}(K_{m,i}-1)\dots(K_{m,i}-q+1)}{\langle K_m \rangle^q} \quad (4)$$

Here  $\langle K_m \rangle = \frac{1}{N_{evts}} \sum_{i=1}^{N_{evts}} K_{m,i}$ . The average content of bin m over the ensemble of events, is substituted for  $\langle \frac{N}{M} \rangle^q$ . In this formulation the factorials are weighted by the average multiplicity for a given bin rather than the overall average bin multiplicity. A correction factor for horizontal moments has been proposed by Fialkowski et al [6] to compensate for the non uniform shape of the rapidity distributions. The horizontal moments are corrected by dividing the factor

$$R_F = \frac{1}{M} \sum_{m=1}^M M^q \frac{\langle K_m \rangle^q}{\langle N \rangle^q} \quad (5)$$

Horizontal moments corrected in this way are compared with the corresponding vertical moments. The correction procedure should be used with some caution, since it is derived under the assumption that the argument of the outer summation in eq.(4) is independent of bin location; this assumption is valid only for relatively small windows in the central region, and substantial variation may occur for windows approaching the target and projectile fragmentation regions. The anomalous fractal dimensions  $d_q$  related to  $\phi_q$  by

$$d_q = \frac{\phi_q}{q-1} \quad (6)$$

Factorial moments and factorial cumulants are claimed to be sensitive to the nature of phase transitions. A  $d_q$  which is independent of q could be taken as an indication of a second order phase transition. In order to cope with the rapidly changing particle densities in the said regions the pseudorapidity variable,  $\eta$ , is transformed into the variable  $\chi$  defined as so that  $\chi$  is uniformly distributed in the interval 0 to 1.

$$\chi(\eta) = \frac{\int_{\eta_1}^{\eta} \rho(\eta') d\eta'}{\int_{\eta_1}^{\eta_2} \rho(\eta') d\eta'} \quad (7)$$

One alternative method of isolating related and the few first orders are given by

$$K_2 = F_2 - 1 \quad (8)$$

$$K_3 = F_3 - 3(F_2 - 1) - 1 \quad (9)$$

Factorial correlators do not only measure the amount of non-statistical fluctuations, but also correlates these fluctuations in different regions of phase space. Here the logarithm of factorial correlators  $\langle F_{pq} \rangle$  defined as

$$\langle F_{pq} \rangle = \frac{1}{M} \sum_{m=1}^M \frac{\langle K_m(K_m-1)\dots(K_m'-q+1) \rangle}{\langle K_m(K_m-1)\dots(K_m'-q+1) \rangle} \quad (10)$$

$$H_q(v) = \sum_{k=0}^{\infty} a_{q,k} v^k \tag{11}$$

This variation of  $\ln \langle F_q \rangle$  are studied as a function of  $X = \delta^{\frac{1}{3}}$ .

TABLE 1. The values  $b_{q,0}$ ,  $b_{q,1}$  and  $b_{q,2}$  for  $q = 2, 3$  and  $4$

$b_{q,k}$	2	3	4
$b_{q,0}$	0.279	2.741	3.147
$b_{q,1}$	3.863	-0.385	0.011
$b_{q,2}$	-1.217	-0.611	-4.040

The data is fit to a polynomial of degree at least two. The sign of coefficient of  $X$  will then speak about the order of phase transition in the Pb-Pb collisions.

### 3. WA98 EXPERIMENT

The WA98 experiment was a fixed target experiment and was geared to look for signals for deconfined state of matter (Quark Gluon Plasma). For the present analysis, data from the Photon Multiplicity Detector (PMD) is used. The focus of the analysis has been to examine the pseudo-rapidity distributions obtained for the  $\gamma$ -like particles in pre-shower photon multiplicity detector in the context of variables connected with Entropy evolution. Only data from the runs in year 1996 has been used in this analysis owing to the better performance of chambers during that run period. The sophisticated online nuclear detector PMD uses scintillating plastics as the radiation sensitive material and lead sheets of three radiation length as the converter for the development of the electromagnetic shower. The readout signal after pedestal correction is subjected to cluster algorithm to count for gamma-like clusters. The output of experiments is the photon multiplicity with pseudorapidity coverage  $2.5 \leq \eta < 4.2$  of which  $3.2 \leq \eta < 4.0$  has full azimuthal angular configurations. The detector has been designed, fabricated and assembled in India and installed at European Nuclear Research Centre (CERN) as a part of WA98 collaboration. The phenomenological models on nuclear collisions are discussed. The theoretical predictions from Quantum chromodynamics for the onset of deconfined state of nuclear matter (QGP).

TABLE 2: Experimental data of WA98 experiment

Data sets	Type	$E_T$ (GeV)	No. of events
Cent-I	Most central	> 347.6	5342
Cent-II	Central	225.5-298.6	5656
Cent-III	Peripheral	89.9-124.3	8748

The results from the analysis of data on photon multiplicities distributions of Pb+Pb collisions at 158 A GeV beam energy are discussed below.

### 4. DETAILS OF CALCULATIONS

The analysis is made on central collisions only which are characterized by ET cut of 348.8 GeV. From the sample of the interactions, we have collected 10000 central events. The quality of the events is tested for various plots like  $\eta$ -distribution and  $\phi$ -distribution which are in line with the published results. For this purpose the VENUS 4.12 and GEANT (GWA98) package is used and the number of central collisions simulated was about 5342. The data is analysed in restricted pseudorapidity window  $3.2 < \eta < 4.0$  with full azimuthal coverage. The distribution of particles in bins of different size ( $\delta\eta=0.1$ ) from a given pseudorapidity. These events were also subjected to all the procedures laid down for the experimental data. A relative comparison of the above studies was made for the relevant parameters and attempt has been made to understand the mechanism of the particle production and the observation of the fluctuations in the photon multiplicities in Pb-Pb collisions at 158 A GeV.

TABLE 3. The values of coefficients of A, B and C obtained from horizontal scaled factorial moment corrected for Experiment central data (polynomial fit).

$b_{q,k}$	2	3	4
$b_{q,0}$	$0.052 \pm 0.02$	$0.035 \pm 0.02$	$0.337 \pm 0.04$
$b_{q,1}$	$0.654 \pm 0.01$	$0.035 \pm 0.12$	$0.443 \pm 0.01$
$b_{q,2}$	$0.048 \pm 0.02$	$0.048 \pm 0.12$	$0.542 \pm 0.01$

TABLE 4. The values of coefficients of A, B and C obtained from horizontal scaled factorial moment corrected for simulated central data (polynomial fit).

$b_{q,k}$	2	3	4
$b_{q,0}$	$0.051 \pm 0.01$	$0.032 \pm 0.03$	$0.547 \pm 0.03$
$b_{q,1}$	$0.041 \pm 0.02$	$0.051 \pm 0.05$	$0.742 \pm 0.05$
$b_{q,2}$	$0.055 \pm 0.01$	$0.046 \pm 0.06$	$0.752 \pm 0.05$

## 5. SUMMARY AND CONCLUSION

Scaled factorial moments of the  $n$  distributions, for windows spanning the central region, have been calculated using both the horizontal and vertical analysis techniques. While the moments exhibit power-law dependence on  $n$  consistent with expectation for intermittent production processes. We find poor evidence for an intermittent pattern of rapidity density fluctuations in Pb-Pb data at 158 A GeV. The origin of such intermittent fluctuations is still unclear and remains an interesting area for both theoretical and experimental studies at LHC energies. To understand the nature of phase transitions in the description of Ginzburg-Landau Model, the Scaled Factorial Moments are utilized. Large multiplicity fluctuations exist with hadronisation process in phase transitions. The study of multiplicity fluctuations of hadrons in final state enables one to find some new quantities to reflect the features of different kinds of phase transitions. The characteristics of Scaled Factorial Moments (SFM) are investigated in the context of Ginzburg-Landau Model and the attempt is made to identify experimental signatures to specify the nature of phase transitions. The behavior of  $\ln F_q$  as a function of resolution

enables one to determine the order of phase transitions unambiguously. The present studies have resulted in useful inferences in regard to parameters discussed above for ultra-relativistic Pb-Pb collisions at 158 A GeV. The experimental results are compared with predictions of simulation studies using Monte Carlo (MC) code, VENUS, for particle production in relativistic nucleus-nucleus collisions.

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